

# Frédéric Chardard

Born July 31st, 1982 at Chatenay-Malabry, France  
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## Home

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## 1 Employment

Since 2009	Agrégé-préparateur (similar to a temporary assistant-professor position) at École Normale Supérieure de Lyon.
2006-2009	“Allocation de recherche” (PhD grant) and “monitorat” (TA position).
2002-2006	Pupil at École Normale Supérieure de Cachan.

## 2 Education

2006-2009	PhD at Centre de Mathématiques et de Leurs Applications, École Normale Supérieure de Cachan (France). Advisor: Frédéric DIAS, Topic: Stability of solitary waves PhD Committee: Thomas J. BRIDGES      Surrey, UK Frédéric DIAS      ENS Cachan      Thesis advisor Christopher K.R.T. JONES      UNC, USA      In the reading committee Juan-Pablo ORTEGA      CNRS Jean-Claude SAUT      Paris XI      President Nikolay TZVETKOV      Lille I      In the reading committee
2003-2005	Master of Science, Partial Differential Equations and Scientific Calculus, Université Paris-Sud XI.

Followed courses:

- Hyperbolic conservation laws and conservation equations ( $2^{nd}$  year).
- Advanced numerical methods for PDEs ( $2^{nd}$  year).
- Geometry ( $1^{st}$  year).
- Evolution problems ( $1^{st}$  year).
- Algebra ( $1^{st}$  year).
- Geometry ( $1^{st}$  year).

2003

Bachelor of Science in Mathematics, Université Paris-Sud XI.

### 3 Teaching

At the master level, I taught:

- Differential geometry course
- Numerical analysis (general background) tutorial
- Finite differences, finite elements, finite volumes tutorial
- Numerical linear algebra tutorial

At the undergraduate level, I taught:

- Numerical analysis tutorial
- Topology tutorial
- Differential calculus tutorial

### 4 Conferences, workshops and seminars

2011/01/12-14	Workshop “Math à Bayonne” on dispersive equations, Bayonne, France.
2010/11/25-26	Partial Differential Equations in Rhône-Alpes Auvergne Days (JERAA), Lyon, France.
2010/06/7-11	Partial Differential Equations Days (JEDP), Port d’Albret, France.
2010/03/02	Partial Differential Equations seminar at at University of Besançon, France.
2009/11/16	MIR@W Day at University of Warwick, United-Kingdom.
2009/11/12-13	Partial Differential Equations in Rhône-Alpes Auvergne Days (JERAA), Grenoble, France.

2009/04/10	Partial Differential Equations seminar at LAMA-University of Savoie.
2009/06/23-26	6 <sup>th</sup> IMACS conference on “Non-linear Evolution Equations and Wave Phenomena: Computation Methods and Theory”, Athens, Georgia, USA.
2009/03/18-19	Workshop of MOAD (Modelisation, Asymptotics, Non-linear Dynamics) research group, Grenoble, France.
2008/09/22-26	Workshop on “Models for fluids, particles and radiation. Confrontation between physical models and numerical models”, Cargèse, France.
2008/07/21-24	SIAM conference on “Non-linear Waves and Coherent Structures”, Roma, Italy.
2008/07/18-19	Workshop on “Multidimensional Localized Structures”, Roma, Italy.
2008/03/19-21	Workshop of MOAD research group, Lyon, France.
2007/06/07-08	SMAI conference on “Applied Mathematics”, Praz-sur-Arly, France.
2006/10/23-27	Workshop on “Current Challenges in Fluid Mechanics”, CIRM, Marseille, France.
2006/09/25-29	Workshop on “Models for fluids, particles and radiation. Confrontation between physical models and numerical models”, Cargèse, France.
2006/09/9-13	SIAM conference on “Non-linear Waves and Coherent Structures”, Seattle, Washington, USA.
2006/09/6-8	Workshop on “Stability and instability of non-linear waves”, Seattle, Washington, USA.
2006/06/14-15	Conference on “Geometry and Mechanics”, University of Surrey, United Kingdom.
2006/09/9-13	Graduate courses on “Nonlinear Waves in PDEs”, University of Surrey, United Kingdom.

## 5 Reviewing activities

I have been a referee for the following journals:

- European Journal of Mechanics - B/Fluids
- European Physical Journal - Applied Physics (EPJAP)

I am also a reviewer for Mathscinet.

## 6 Publications

Published articles:

- F.CHARDARD, *Maslov index for solitary waves obtained as a limit of the Maslov index for periodic waves*, C. R. Acad. Sci. Paris, Ser. I **345/12** (2007), 689–694.

- F.CHARDARD, F.DIAS, T.J.BRIDGES, *Fast computation of the Maslov Index for hyperbolic linear systems with periodic coefficients*, Journal of Physics A : Mathematical and General **39** (2006), no. 47, 14545–14557.
- F. CHARDARD, F. DIAS & T.J. BRIDGES. *On the Maslov index of multi-pulse homoclinic orbits*, Proc. Royal Soc. London A **465** 2897–2910 (2009).
- F. CHARDARD, F. DIAS & T. J. BRIDGES, *Computing the Maslov index of solitary waves. part 1: Hamiltonian systems on a 4-dimensional phase space*, Physica D **238** (2009), no. 18, 1841–1867.
- F.CHARDARD, F.DIAS, H.Y. NGUYEN, J.M. VANDEN-BROECK, *Stability of some stationary solutions to the forced KdV equation with one or two bumps*, Journal of Engineering Mathematics, **70** (2011), 175–189.

Under press:

- F.CHARDARD, F.DIAS, T.J.BRIDGES, *Computing the Maslov index of solitary waves. Part 2: Hamiltonian systems on a 2n-dimensional phase space*, accepted in Physica D.

## 7 Summary of accomplished works

These works have been done during my thesis at ENS Cachan and since I am Agrégé-Préparateur at ENS Lyon. They have been devoted to the stability of periodic and solitary waves, and in particular to the application of the Maslov index theory to the spectral stability problem. In the following summary, we first show the relevance of Maslov index theory on the example of the Kawahara equation and describe how the Maslov index of solitary waves is related to the Maslov index of periodic orbits. Then we describe how to compute the Maslov index, for low and high dimensional systems. Lastly, we deal with the stability of stationary solutions of a model for flows over a non-uniform bottom.

### An example where the Maslov index arises: Kawahara equation

Consider Kawahara equation written in a frame moving at speed  $c$ :

$$\frac{\partial u}{\partial t} - c \frac{\partial u}{\partial x} + \frac{\partial}{\partial x} (u^{q+1}) + P \frac{\partial^3 u}{\partial x^3} - \frac{\partial^5 u}{\partial x^5} = 0, \quad q \geq 1. \quad (1)$$

This is a model for plasmas and capillarity-gravity water-waves (see [10]).

This equation has a Hamiltonian structure: it can be put under the form  $u_t = \mathcal{J} \nabla H$ , where  $\mathcal{J} = \frac{\partial}{\partial x}$  is a skew-symmetric operator and  $H(u) = \int_{\mathbb{R}} \frac{1}{2} u_{xx}^2 + \frac{P}{2} u_x^2 - \frac{1}{q+2} u^{q+2} + \frac{c}{2} u^2 dx$  the Hamiltonian.

A solitary wave  $\phi$  is a stationary solution  $\phi$  of (1) exponentially decaying at  $\pm\infty$ .  $\phi$  is also a critical point of the Hamiltonian.

According to [5, 4], the spectral stability is linked to the number  $l$  of strictly negative eigenvalues of the hessian  $\mathcal{L}u = (u_{xxxx} - P u_{xx} - (q+1)\phi^q u + cu)$  of  $H$  at  $\phi$ :

$$l = \#\{\text{Oscillatory eigenmodes with negative energy of } \partial_x \mathcal{L}\} + \#\{\text{Unstable eigenmodes of } \partial_x \mathcal{L}\} + r$$

with  $r = \begin{cases} 1 & \text{if } \int_{\mathbb{R}} \phi \psi > 0 \\ 0 & \text{if } \int_{\mathbb{R}} \phi \psi < 0 \end{cases}$  and  $\psi$  such that  $\mathcal{L}\psi = \phi$ .

The spectral problem  $\mathcal{L}u = \mu u$  is equivalent to a finite-dimensional Hamiltonian linear system:

$$U_x = JA(x, \mu)U, \quad (2)$$

with  $J = \begin{pmatrix} 0 & -I_n \\ I_n & 0 \end{pmatrix}$  and  $A$  symmetric (here,  $n = 2$ ,  $U = \begin{pmatrix} u \\ u_{xx} \\ u_{xxx} - Pu_x \\ = u_x \end{pmatrix}$  and  $A(x, \lambda) =$

$$\begin{pmatrix} 1 - (p+1)\widehat{\phi}(x)^q - \lambda & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & P \end{pmatrix}).$$

For problems of that kind, it is possible to use Maslov index theory to count the eigenvalues of  $\mathcal{L}$ .

### The Maslov index of solitary waves

While Maslov index theory for periodic systems has been well-studied because of its applications ranging from semiclassical quantization, quantum chaology, stability of waves and classical mechanics, this is not the case of the Maslov index for solitary waves. The latter was first introduced in [2, 9] and later in [3].

A Maslov index for solitary waves can be defined by approximating the solitary wave with periodic waves: when a sequence of periodic waves  $\phi^\alpha$  converges to the solitary wave  $\phi$ , we proved that the sequence of Maslov indices converges and its limit can be used as a definition for the Maslov index of  $\phi$ .

The limit, called the Maslov index  $I(\mu)$  of system (2), exists whenever  $\mu$  is not in the spectrum of  $\mathcal{L}$ : the space of solutions that decay to 0 at  $-\infty$ , called the unstable subspace, is a closed loop in the manifold of Lagrangian subspaces (A subspace  $E$  is said to be Lagrangian if it has dimension  $n$  and if  $\forall x, y \in E, {}^T x J y = 0$ ). Since this manifold has homotopy group  $\mathbb{Z}$ , it is possible to attribute an integer to each closed path.  $I(\mu)$  is equal to the integer associated to the path made by the unstable space.  $I(\mu)$  is also equal to the number of eigenvalues of  $\mathcal{L}$  below  $\mu$ . The case where  $\mu$  is in the discrete spectrum has also been studied, but is a more difficult to deal with and there is not necessarily convergence.

### Computing the Maslov index numerically

We developed an algorithm where we compute on the exterior algebra of the ambient vector space. Any subspace can indeed be represented by an element of exterior algebra, unique up to a multiplication by a scalar (see [1] for example). This approach eliminates stiffness. This algorithm enabled us to compute the Maslov index of a solitary wave. We also described an algorithm adapted to the periodic case.

We applied this algorithm to Longwave-Shortwave resonance equations (see [11] for a description of these equations) and to the aforementioned Kawahara equation. For some values of  $P$  and  $c$ , Kawahara equation admits an infinity of solitary wave solutions, each made of several “pulses” separated by a variable distance. These multi-pulse homoclinic orbits can be classified by a sequence of integers.

We have found the surprising result that this string of integers encodes the value of the Maslov index of the homoclinic orbit.

We also studied the change in the Maslov index when we move along a branch of solutions as the parameter  $P$  is varied.

Unfortunately, exterior algebra is only usable when  $n$  is low since the dimension of exterior algebra grows exponentially with  $n$ . So we developed a numerical framework based on discrete orthogonalisation. A Lagrangian space is now described by a  $2n \times n$ -matrix  $\begin{pmatrix} V \\ W \end{pmatrix}$  whose columns span the Lagrangian space. This representation is unique up to a multiplication by a  $n \times n$ -matrix on the right. In this algorithm, when integrating numerically equation (2) is needed,  $\begin{pmatrix} V \\ W \end{pmatrix}$  is rescaled at each space step in a way such that  $V + iW$  is unitary.

We tested this algorithm on the exact solitary waves of a seventh-order Korteweg-de Vries equation described in [8].

### Instability of some steady free-surface flows past submerged obstacles

An other topic which we studied is the steady flows which may arise in the neighborhood of an obstacle.

We used the following forced Korteweg-de Vries model, which had already been used in [12, 7]:

$$\eta_t = \frac{1}{6}\eta_{xxx} + \frac{3}{4}(\eta^2)_x - (F - 1)\eta_x + \frac{1}{2}B_x, \quad (3)$$

$\eta$  is the height of the free surface of water and  $B$  is the bottom of the fluid.

For example, we showed that two distant obstacles could generate a table-top type solution:

$$\eta = A \tanh(\alpha(x + L)) - A \tanh(\alpha(x - L)), \quad A = \frac{2}{3}(F - 1)$$

We proved that at least an infinity of these solutions were unstable. This can be done by using Sturm-Liouville theory which is the one-dimensional case of Maslov index theory.

In addition to these solutions, we also studied numerically the case where the perturbation on the free surface is localized near the obstacles, by integrating system (3) forward in time, using a Fourier-type space discretization.

We also studied numerically the stability of hydraulic fronts generated by a single obstacle:

$$\eta = \frac{2}{3}(F - 1)(1 + \tanh(\alpha x))$$

In that case, periodic boundary conditions are not suitable anymore and we had to use the scheme of lower-order described in [6]. It turns out that when  $\alpha > 0$ , i.e. when the water actually rises when going from left to right, the stationary solution is unstable.

### Conclusion and perspectives

The Maslov index is an other topological tool which enables to relate the shape of a solitary wave to its stability. To clarify the behaviour of the Maslov index near bifurcation points, we also intend to introduce a transversality index and to relate it to the parity of the Maslov index. Concerning the flow past several obstacles, there are a lot of solutions whose stability has to be determined.

### References

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