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Ph.D in mathematics

Background:

In my thesis, I have studied semisimple complex Lie algebras and a generalization of those objects: symmetric (semisimple complex) Lie algebras. Let us recall that a symmetric Lie algebra is a pair $(\mathfrak{g}, \theta)$ where $\mathfrak{g}$ is a Lie algebra and θ an automorphism of $\mathfrak{g}$. In this setting, one can decompose $\mathfrak{g}$ into +1 and -1 eigenspaces respectively denoted by $\mathfrak{k}$ and $\mathfrak{p}$: $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{p}$. Let G stand for the adjoint group of $\mathfrak{g}$ and K for the connected subgroup of G with Lie algebra $\mathfrak{k}$. Then, $\mathfrak{p}$ is an K-variety and the G-variety $\mathfrak{g}$ can be seen as a particular case of such variety.

So far, I have studied two types of subvarieties arising in this context. The first one is the *symmetric nilpotent commuting variety*

$$\{(x, y) \in \mathfrak{p} \times \mathfrak{p} \mid [x, y] = 0 \text{ and x,y are nilpotent } \},$$

and the second one is sheets which are the irreducible components of subset of the form

$$\mathfrak{p}^{(m)} := \{x \in \mathfrak{p} \mid \dim K.x = m\}, \quad m \in \mathbb{N}.$$

I managed to prove that the symmetric nilpotent commuting variety is equidimensionnal in a significant amount of cases *via* a good parametrization. Concerning sheets, I have been able to parametrize them under some reasonable assumptions which are satisfied when $\mathfrak{g} = \mathfrak{sl}_n$. Furthermore, I shew that they are smooth.

Research plan:

Direct continuation of thesis is possible. First of all, it is natural to ask whether results obtained for the nilpotent commuting variety and sheets remain true in other cases. This should require additional techniques to those used in my thesis. In this way, some results such as the parametrization of K-sheets should be reachable in more classical cases.

In addition, it could be interesting to improve our knowledge about other aspects of these varieties. For instance, one can hope proving that K-sheets have a geometric quotient, at least in the case $\mathfrak{g} = \mathfrak{gl}_n$. As moduli spaces is a subject of great interest in MPIM, I hope to improve my knowledge in this area. As an other example, one can study the ideal of definition of the nilpotent commuting variety and try to relate the question of its reduceness to the one of the commuting variety, the last one being still an open question.

Some other varieties can also be studied. Recently (2007), Vladimir L. Popov has given some results about the singular locus (*i.e.* the set of elements whose stabilizer in G is not of minimal dimension) of the commuting variety of Lie algebras. This has provided information on the normality and the reduceness question of the commuting variety. It should be very interesting to try to adapt his methods to the symmetric case, which is a bit more involved. The nilpotent bicone $\{(x, y) \mid \langle x, y\rangle \subset \mathcal{N}(\mathfrak{g})\} \subset \mathfrak{g} \times \mathfrak{g}$ described by Anne Moreau (2008) also has a natural analogue in the symmetric case. Methods used in the Lie algebra case should not be easily translated to the symmetric setting. Therefore, one should try to develop more techniques in order to get information about the symmetric nilpotent bicone.

A far-reaching goal, which could justify such above-mentioned studies, would be a better understanding of the diagonal actions of G on $\mathfrak{g} \times \mathfrak{g}$ and K on $\mathfrak{p} \times \mathfrak{p}$. The first one is related to the study of the quiver L_2 in the case $\mathfrak{g} = \mathfrak{gl}_n$ which is a wild quiver. This *doubled setting* is far from being well understood. For example, one do not know any good moduli space of $\mathfrak{gl}_n \times \mathfrak{gl}_n$ under GL_n. Several works also try to find out good *double* analogues of nilpotent elements, such as the principal nilpotent commuting pairs of Ginzburg (2001). A global understanding of the doubled setting should take advantage of lots of these notions and their properties.