Augmented Lagrangian without alternating directions: practical algorithms for inverse problems in imaging
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Starting point
Many image processing tasks are formulated as a large scale optimization problem:

- image restoration
- image segmentation

Typical optimization problems in imaging

- convex
- smooth
- non-smooth
- e.g., L1 norm to promote sparsity or indicator function for constraints

Results

- image deconvolution [Matakos, Ramani & Fessler 2013]

- image segmentation [Houhou, Thiran & Bresson 2009]
  [Goldstein, Bresson & Osher 2010]

  \[
  \arg \min_{x \geq 0} \| H x - y \|_W^2 + \lambda TV(x)
  \]

  with \( h \) defined from textural descriptors

- image segmentation [Houhou, Thiran & Bresson 2009]
  [Goldstein, Bresson & Osher 2010]

  \[
  \arg \min_{x \leq x \leq 1} h^T x + \lambda TV(x)
  \]

Hierarchical optimization

our proposition

- \( z^* \) can generally be computed in closed-form
- \( z^* \) can be solved approximatively using a quasi-Newton method with \( \nabla = \nabla f + u(x - z^*) \)
- \( u \leftarrow u + \rho (x - z) \)

ADMM
Alternating directions method of multipliers

- works best with multiple splittings so that each sub-problem can be solved exactly
- [Matakos, Ramani & Fessler 2013] requires fine-tuning the parameter(s) \( \rho \)

The augmented Lagrangian reformulation

\[
\arg \min_{x, u, \lambda} f(x) + u(x - z^*) + \frac{\rho}{2} \| x - z^* \|_2^2
\]

\[
\text{subject to } x = z
\]

- the optimization problem is too hard \( \rightarrow \) separate variables
- turn the constrained problem into an unconstrained form with the augmented Lagrangian:

- limited-memory quasi-Newton methods are very efficient
- cannot apply to non-smooth problems
- many algorithms for non-smooth optimization
- can be very slow

no parameter tuning

reaches state-of-the-art performance with