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After writing the paper [Nuc10], I became aware of the paper [FK86] where several examples of real quadratic fields verifying Greenberg conjecture are presented, as an application of Fukuda and Komatsu's main theorem. In particular, all pairs $\big(k = \mathbb{Q}(\sqrt{m}), p\big)$ of real quadratic fields and primes listed on page 241/242 right after Proposition 2 verify

- p splits in k ;
- Greenberg's conjecture holds for (k, p)
- The subgroup of the p-Sylow of Cl_k generated by classes of primes above p is non-trivial (it is the subgroup denoted by D_0 in [FK86] and by Π_0 in [Nuc10]).

The proof of Proposition 2 in [FK86] shows that $D_n = B_n = B_0 = D_0$ for all $n \geq 0$, so the subgroup denoted by Π_n in [Nuc10] is non-trivial for every n. Thanks to Corollary 5.2 of *ibid.*, this shows that the kernel and cokernel of the exact sequence of Theorem 1 in *ibid.* are non-trivial.

The mentioned list is as follows (I simply write m for the field $k = \mathbb{Q}(\sqrt{m})$:

p	m
3	142, 223, 229, 235, 346, 427, 469, 574, 697, 895, 898, 934, 985, 1090, 1171, 1342, 1345, 1489, 1495, 1522, 1567, 1627, 1639, 1735, 1765, 1771, 1957, 1987
5	401, 439, 499, 1126, 1226, 1429, 1486, 1766, 2031, 2081, 2986, 3121, 3129, 3134, 3181, 3246, 3379, 3599, 3601, 3814, 3966, 4271, 4321, 4334, 4359, 4591, 4889
7	2251, 2599, 2913, 3595, 3679, 4139, 4229, 4579

References

[FK86] Takashi Fukuda and Keiichi Komatsu, *On the λ invariants of $\mathbf{Z}_p$-extensions of real quadratic fields*, J. Number Theory **23** (1986), no. 2, 238–242.

[Nuc10] Filippo A. E. Nuccio, *Cyclotomic units and class groups in $\mathbb{Z}_p$-extensions of real abelian fields*, Math. Proc. Cambridge Philos. Soc. **148** (2010), no. 1, 93–106.