Erratum to “Koszulity for nonquadratic algebras”

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In Section 5, just below Lemma 5.5, it is not true that the differentials d'_L and d'_R over K_{L-R} commute if $s \geq 3$. The differential d' has to be changed in the following manner. Introduce $(\overline{K}_L, \delta_L)$ by $\overline{K}_{L,n} = A \otimes J_n$, $n \geq 0$, and the A-linear map $\delta_L : \overline{K}_{L,n} \to \overline{K}_{L,n-1}$ is defined by the natural inclusion $J_n \hookrightarrow A \otimes J_{n-1}$. We have $(\delta_L)^s = 0$, so $(\overline{K}_L, \delta_L)$ is a s-complex. Define analogously the s-complex $(\overline{K}_R, \delta_R)$. Then $\overline{K}_{L-R} = \overline{K}_L \otimes A = A \otimes \overline{K}_R$ is a bimodule s-complex for $\delta'_L = \delta_L \otimes 1_A$ (respectively, for $\delta'_R = 1_A \otimes \delta_R$). But now δ'_L and δ'_R commute. So the good differential d' over K_{L-R} is defined by

$$d'_i = \delta'_L - \delta'_R,$$

if i is odd, and

$$d'_i = \delta'^{s-1}_L + \delta'^{s-2}_L \delta'_R + \ldots + \delta'_L \delta'^{s-2}_R + \delta'^{s-1}_R,$$

if i is even. We obtain a pure projective complex in the category $\mathcal{C}$, called the bimodule Koszul complex of A. The exact sequence (5.6) still holds, as well Theorem 5.6 with the same proof.

Formulas (5.12) and (5.13) have to be replaced accordingly. Assuming i odd ≥ 3, we have

$$\tilde{d}_i(a \otimes vwv') = av \otimes wv' - \ v'a \otimes vw,$$

where v and v' are in V, and w is in $V^{(js-1)}$. Assuming i even, we have

$$\tilde{d}_i(a \otimes v_1 \ldots v_{js}) = av_1 \ldots v_{s-1} \otimes v_s \ldots v_{js} + v_{js} a v_1 \ldots v_{s-2} \otimes v_{s-1} \ldots v_{js-1}$$

$$+ \cdots + v_{js-s+2} \ldots v_{js} a \otimes v_1 \ldots v_{js-s+1},$$

where $v_1, \ldots, v_{js}$ are in V.

While (5.16) still holds, (5.15) is replaced by a sum of s appropriate sums, each of those is indexed by $Q \subset P$, $|Q| = s-1$. In particular, $s[P]$ takes place of $2[P]$ in (5.23). All the remainder is unchanged.